Name: ______________________ **Date:** ___________________

$\sqrt[3]{54}$ entire

mixed

Math 10F&PC Chapter 4 Roots and Powers

4.3 Mixed and Entire Radicals

$\sqrt[3]{54} = \sqrt[3]{27}\cdot\sqrt[3]{2} = 3\sqrt[3]{2}$

Expressing an ***entire radical*** *as a* ***mixed radical*** *and vice versa is similar to the concept of writing equivalent fractions.*

$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

Just like $\frac{1}{2}$ is equivalent to $\frac{4}{8}$ the radical $\sqrt{8}$ is equivalent to $\sqrt{4}\cdot\sqrt{2}$ equivalent to $2\sqrt{2}$.

Entire $\sqrt{8} = \sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$ → mixed

And $\sqrt{25\times 9}$ is the same as $\sqrt{25}\times\sqrt{9}$ because $5\cdot 3 = \boxed{15}$

$\sqrt{225} = \boxed{15}$ $\quad 5\times 3 = \boxed{15}$

Entire Radical*: a radical in the form* $\sqrt{x}$ $\quad$ *Examples*: $\sqrt{35}$, $\sqrt{68}$, $\sqrt[3]{47}$, $\sqrt[4]{39}$

Mixed Radical: *a radical in the form* $a\sqrt{x}$ $\quad$ *Examples*: $3\sqrt{5}$, $-4\sqrt{21}$

The process for creating equivalent fractions is multiplying by a common number to both the top and bottom of the fraction. The process of writing equivalent radicals or writing an entire radical as a mixed radical, otherwise known as ***simplifying*** the radical involves the multiplication property of radicals:

$\frac{3}{7} \overset{\times 2}{=} \frac{6}{14} = \frac{9}{21} \cdots\cdots$ $\quad$ ×2 $\quad$ ×3

Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a}\times\sqrt[n]{b}$$

Where *n* is a natural number, and *a* and *b* are real numbers.

$\sqrt[3]{35} = \sqrt[3]{7}\cdot\sqrt[3]{5}$

Part 1: Entire Radicals to Mixed Radicals

1. *Find the* ***LARGEST*** *perfect square that is a factor of the radicand*
2. *Rewrite the radicand as a product of its largest square and another number.*
3. *Take the square root of the perfect square and write the answer as a mixed radical*

Example 1:

a) $\sqrt{72} = \sqrt{36}\cdot\sqrt{2} = 6\sqrt{2}$

b) $\sqrt{63} = \sqrt{9}\cdot\sqrt{7} = 3\sqrt{7}$

c) $\sqrt[3]{108} = \sqrt[3]{2\cdot 2\cdot 3\cdot 3\cdot 3} = 3\ \boxed{3\sqrt[3]{4}}$

d) $\sqrt[4]{128} = \sqrt[4]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2} = \boxed{2\sqrt[4]{8}}$

e) $\sqrt{80}$

$\sqrt{80} = \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 5} = 2\cdot 2\sqrt{5} = \boxed{4\sqrt{5}}$

f) $\sqrt[4]{162} = 3\sqrt[4]{2}$

$\sqrt[4]{162} = \sqrt[4]{2\cdot 3\cdot 3\cdot 3\cdot 3} = \boxed{3\sqrt[4]{2}}$

Some number, such as 252, have more than one perfect square factor. The factors of 252 are: 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252. Since 4, 9, and 36 are perfect squares, we can simplify 252 in three ways.

252 = 2 · 126, 126 = 2 · 63, 63 = 3 · 21, 21 = 3 · 7

a) $\sqrt{\cancel{256}}$ $\sqrt{252}$
$= \sqrt{4}\cdot\sqrt{63}$
$= 2\sqrt{63}$

b) $\sqrt{\cancel{256}}$ $\sqrt{252}$
$= \sqrt{9}\cdot\sqrt{28}$
$= 3\sqrt{28}$

c) $\sqrt{\cancel{256}}$ $\sqrt{252}$
$= \sqrt{36}\cdot\sqrt{7}$
$= 6\sqrt{7}$
$= \sqrt{2\cdot2\cdot3\cdot3\cdot7} = \boxed{6\sqrt{7}}$

$6\sqrt{7}$ is in simplest form because the radical contains no perfect square factors other than 1. To write a radical of index *n* in simplest form, we write the radicand as a product of 2 factors, one of which is the greatest perfect *n*th power.

More mixed radicals that can be further reduced

27 = 3 · 9, 9 = 3 · 3

a) $6\sqrt{27} = 6\sqrt{3\cdot3\cdot3}$
$= 6\times3\sqrt{3} = \boxed{18\sqrt{3}}$

b) $4\sqrt{20} = 4\sqrt{4}\cdot\sqrt{5}$
$= \boxed{8\sqrt{5}}$

c) $5\sqrt{80}$ ($\sqrt{80} = 4\sqrt{5}$)
$= 5\times4\sqrt{5} = \boxed{20\sqrt{5}}$

Part 2: Mixed Radicals to Entire Radicals

1. *Square the number outside of the radical and write it under a radical sign.*
2. *Multiply the two numbers together and put the answer under a radical sign.*

a) $3\sqrt{6} = \sqrt{6\cdot3\cdot3}$
$= \sqrt{54}$

b) $4\sqrt{5} = \sqrt{5\cdot4\cdot4}$
$= \sqrt{80}$

b) $6\sqrt{27} = \sqrt{27\cdot6\cdot6}$
$= \sqrt{972}$

d) $2\sqrt[3]{2} = \sqrt[3]{2\cdot2\cdot2\cdot2}$
$= \sqrt[3]{16}$

e) $5\sqrt{2} = \sqrt{2\cdot5\cdot5}$
$= \sqrt{50}$

f) $6\sqrt[4]{5} = \sqrt[4]{5\cdot6\cdot6\cdot6\cdot6}$
$= \sqrt[4]{6480}$

Simplifying radicals. (Divide)

a. $\dfrac{\sqrt{10}}{\sqrt{2}} = \dfrac{\cancel{\sqrt{2}}\cdot\sqrt{5}}{\cancel{\sqrt{2}}} = \boxed{\sqrt{5}}$

$= \dfrac{5\sqrt{2}}{\sqrt{2}}$
$= \cancel{5}$

b. $\dfrac{\sqrt{6}}{\sqrt{2}} = \dfrac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{2}}$
$= \boxed{\sqrt{3}}$

c. $\dfrac{\sqrt{54}}{\sqrt{2}} = \dfrac{\sqrt{2}\cdot\sqrt{27}}{\sqrt{2}}$
$= \sqrt{27} = \boxed{3\sqrt{3}}$

Assignment: ***p. 218 #4, 5, 9–12, 15–18,20–22*** Quiz next day